

Operacje arytmetyczne na  
liczbach dwójkowych.  
Dodawanie dwójkowe

Zasady dodawania, czyli wyniki sumowania poszczególnych liczb.

Tabliczka dodawania binarnego

$$0 + 0 = \mathbf{0}$$

$$0 + 1 = \mathbf{1}$$

$$1 + 0 = \mathbf{1}$$

$$1 + 1 = \mathbf{0} \text{ i } \mathbf{1} \text{ dalej}$$

# Dodawanie liczb dwójkowych

$$(1011011)_2 + (101100)_2$$

		<b>1</b>		<b>1</b>		<b>1</b>				
		1	0	1	1	0	1	1		
+			1	0	1	1	0	0		
	1	0	0	0	0	1	1	1		

$$1 + 0 = 1 \quad + \mathbf{1} \text{ z przeniesienia}$$

$$0 + 1 = 1 \quad + \mathbf{1} \text{ z przeniesienia}$$

## Tabliczka dodawania binarnego

$$0 + 0 = \mathbf{0}$$

$$0 + 1 = \mathbf{1}$$

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$$1 + 1 = \mathbf{0} \text{ i } \mathbf{1} \text{ dalej}$$

Operacje arytmetyczne na  
liczbach dwójkowych.  
odejmowanie

# Zasady odejmowania, czyli wyniki różnicy poszczególnych liczb.

## Tabliczka odejmowania binarnego

$$0 - 0 = \mathbf{0}$$

$0 - 1 = \mathbf{1}$  i pożyczka do następnej pozycji

$$1 - 0 = \mathbf{1}$$

$$1 - 1 = \mathbf{0}$$

Pożyczka oznacza konieczność odjęcia 1 od wyniku odejmowania cyfr w następnej kolumnie.













$$1101110_{(2)} - 1111_{(2)}$$

The diagram illustrates the subtraction of  $1111_{(2)}$  from  $1101110_{(2)}$ . The minuend is  $1101110$  and the subtrahend is  $001111$ . A blue arrow indicates a borrow of 1 from the 6th bit of the minuend to the 5th bit. The resulting digits are  $011111$ .

$$\begin{array}{r} \phantom{1}11111 \\ 1101110 \\ - 001111 \\ \hline 011111 \end{array}$$

$1-0 = 1$  od tego odejmujemy  $1$  z pożyczki  $= 0$

$$1101110_{(2)} - 1111_{(2)}$$

$$\begin{array}{r} \phantom{110}1111 \\ 1101110 \\ - \phantom{110}1111 \\ \hline 1011111 \end{array}$$

$$1-0 = 1$$

Przykłady do rozwiązania:

$$\begin{array}{r} 1111111 \\ 10000000 \\ - 00000001 \\ \hline 01111111 \end{array}$$

$$\begin{array}{r} 1111 \\ 11110000 \\ - 00001111 \\ \hline 11100001 \end{array}$$

Operacje arytmetyczne na  
liczbach dwójkowych. Mnożenie

Zasady mnożenia, czyli wyniki iloczyn  
poszczególnych liczb.

Tabliczka mnożenia binarnego

$$0 \times 0 = \mathbf{0}$$

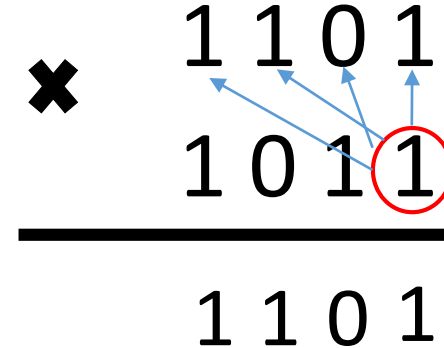
$$0 \times 1 = \mathbf{0}$$

$$1 \times 0 = \mathbf{0}$$

$$1 \times 1 = \mathbf{1}$$

# Mnożenie liczb dwójkowych

np.  $1101_{(2)} \times 1011_{(2)}$

$$\begin{array}{r} \times \quad 1101 \\ \quad 1011 \\ \hline 1101 \end{array}$$
The diagram illustrates the multiplication of two 4-bit binary numbers: 1101 and 1011. The numbers are aligned to the right. A horizontal line is drawn under the second number. Below the line, the result 1101 is shown. Blue arrows indicate the bit-by-bit multiplication process: the rightmost '1' of the second number is multiplied by each bit of the first number, and the results are shifted to the left. The rightmost '1' of the second number is circled in red. The final result 1101 is shown below a horizontal line.

## Tabliczka mnożenia binarnego

$0 \times 0 = 0$

$0 \times 1 = 0$

$1 \times 0 = 0$

$1 + 1 = 1$



# Mnożenie liczb dwójkowych

np.  $1101_{(2)} \times 1011_{(2)}$

$$\begin{array}{r} \times \quad 1101 \\ \quad 1011 \\ \hline \quad 1101 \\ 1101 \end{array}$$

## Tabliczka mnożenia binarnego

$0 \times 0 = 0$

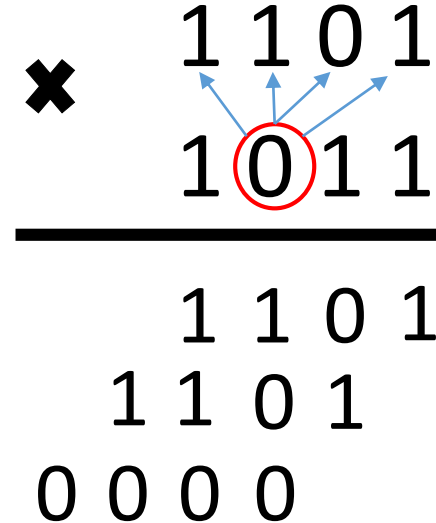
$0 \times 1 = 0$

$1 \times 0 = 0$

$1 + 1 = 1$

# Mnożenie liczb dwójkowych

np.  $1101_{(2)} \times 1011_{(2)}$

$$\begin{array}{r} \times \quad 1101 \\ \quad 1011 \\ \hline \quad 1101 \\ \quad 1101 \\ 0000 \end{array}$$


## Tabliczka mnożenia binarnego

$0 \times 0 = 0$

$0 \times 1 = 0$

$1 \times 0 = 0$

$1 \times 1 = 1$

# Mnożenie liczb dwójkowych

np.  $1101_{(2)} \times 1011_{(2)}$

$$\begin{array}{r} \times \quad 1101 \\ \quad 1011 \\ \hline \quad 1101 \\ \quad 1101 \\ \quad 0000 \\ 1101 \end{array}$$

## Tabliczka mnożenia binarnego

$0 \times 0 = 0$

$0 \times 1 = 0$

$1 \times 0 = 0$

$1 + 1 = 1$

# Mnożenie liczb dwójkowych

np.  $1101_{(2)} \times 1011_{(2)}$

$$\begin{array}{r} \phantom{000} \times \phantom{00} 1101 \\ \phantom{000} \phantom{00} 1011 \\ \hline 0001101 \\ 0011010 \\ + 0000000 \\ 1101000 \\ \hline \end{array}$$

$$\begin{array}{r}
 \times \quad 1101 \\
 \hline
 \phantom{000}1101 \\
 + \phantom{000}0011 \\
 \hline
 0001101 \\
 0011010 \\
 0000000 \\
 1101000 \\
 \hline
 \phantom{000000}1
 \end{array}$$

The diagram shows a binary addition problem. The numbers 1101 and 0011 are added together. The result is 1001101. The carry bit 1 is shown below the result. Blue arrows on the right point to the 1s in the result, with the label 1+0=1.

Tabliczka dodawania binarnego

0 + 0 = **0**

0 + 1 = **1**

1 + 0 = **1**

1 + 1 = **0** i **1** dalej

$$\begin{array}{r}
 \times \quad 1101 \\
 \hline
 \phantom{000}1101 \\
 + \phantom{000}0000 \\
 \hline
 0001101 \\
 0011010 \\
 0000000 \\
 1101000 \\
 \hline
 \phantom{000}11
 \end{array}$$

Diagram illustrating binary addition. The numbers 1101 and 0000 are added. The result is 1101000. Blue arrows point from the result to the equations:
 

- 0+1=1 (pointing to the 1 in the 2<sup>0</sup> place)
- 1+0=1 (pointing to the 1 in the 2<sup>1</sup> place)
- 1+0=1 (pointing to the 1 in the 2<sup>2</sup> place)
- 1+0=1 (pointing to the 1 in the 2<sup>3</sup> place)

Tabliczka dodawania binarnego

0 + 0 = **0**

0 + 1 = **1**

1 + 0 = **1**

1 + 1 = **0** i **1** dalej

$$\begin{array}{r}
 \times \quad 1101 \\
 \hline
 \phantom{00}001101 \\
 + \phantom{00}001101 \\
 \hline
 \phantom{00}000000 \\
 + \phantom{00}110100 \\
 \hline
 \phantom{00}111
 \end{array}$$

Diagram illustrating binary addition. The numbers 001101 and 001101 are added. The result 000000 is shown, with a carry of 1 to the next column. The final result 111 is shown below the horizontal line. Blue arrows and text labels indicate the carry propagation:
 

- 1+1=1 (Carry from the 6th column to the 5th)
- 1+0=1 (Carry from the 5th column to the 4th)
- 1+0=1 (Carry from the 4th column to the 3rd)

Tabliczka dodawania binarnego

0 + 0 = **0**

0 + 1 = **1**

1 + 0 = **1**

1 + 1 = **0** i **1** dalej

$$\begin{array}{r}
 \times \quad 1101 \\
 \hline
 1011 \\
 \hline
 0001101 \\
 0011010 \\
 0000000 \\
 1101000 \\
 \hline
 1111
 \end{array}$$

Diagram illustrating binary addition. The numbers are aligned as follows:
   
 1101 (multiplier)
   
 1011 (multiplicand)
   
 0001101 (product of 1011 and 1)
   
 0011010 (product of 1011 and 0)
   
 0000000 (product of 1011 and 0)
   
 1101000 (product of 1011 and 1)
   
 The sum is 1111.
   
 Annotations:
 

- 1 (green) above the third column of the first row.
- Blue arrows pointing from the 1 in the third column of the first row to the 0 in the fourth column of the second row.
- Blue arrows pointing from the 1 in the fourth column of the second row to the 0 in the fifth column of the third row.
- Blue arrows pointing from the 1 in the fifth column of the third row to the 0 in the sixth column of the fourth row.
- Blue arrows pointing from the 1 in the sixth column of the fourth row to the 0 in the seventh column of the fifth row.

1+1=0 i jeden dalej  
 0+0=0  
 0+1=1

### Tabliczka dodawania binarnego

$$0 + 0 = \mathbf{0}$$

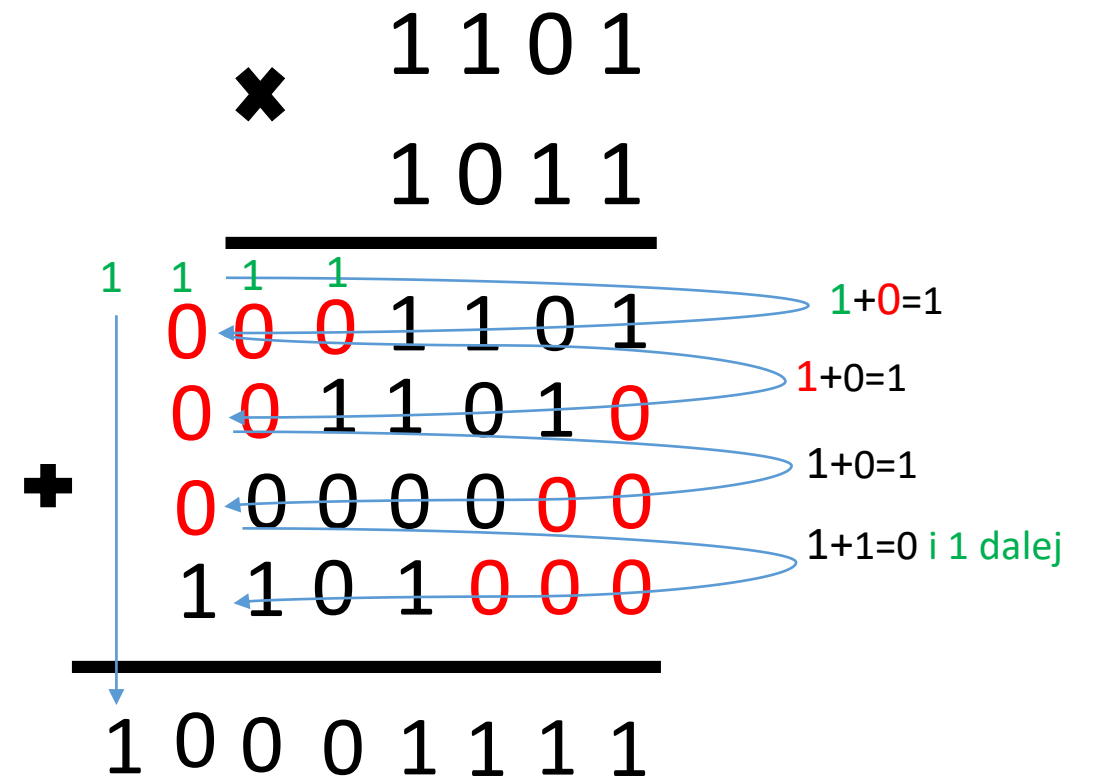
$$0 + 1 = \mathbf{1}$$

$$1 + 0 = \mathbf{1}$$

$$1 + 1 = \mathbf{0} \text{ i } \mathbf{1} \text{ dalej}$$







Tabliczka dodawania binarnego

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 0$  i  $1$  dalej

Wynik

$$\mathbf{1101}_{(2)} \times \mathbf{1011}_{(2)} = \mathbf{10001111}_{(2)}$$

# Przykłady do rozwiązania:

$$101 \times 111$$

$$\begin{array}{r} 101 \\ \times 111 \\ \hline 101 \\ 101 \\ +101 \\ \hline 100011 \end{array}$$

$$1011 \times 110$$

$$\begin{array}{r} 1011 \\ \times 110 \\ \hline 0000 \\ 1011 \\ +1011 \\ \hline 1000010 \end{array}$$

$$111 \times 111$$

$$\begin{array}{r} 111 \\ \times 111 \\ \hline 111 \\ 111 \\ +111 \\ \hline 110001 \end{array}$$